

From Prototype $SU(5)$ to Realistic $SU(7)$ SUSY GUT

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Abstract

We construct a realistic $SU(7)$ model which provides answers to many questions presently facing the prototype $SU(5)$ SUSY GUT. Among them are a solution to the doublet-triplet splitting problem, string scale unification, proton decay, the hierarchy of baryon vs lepton number violation and neutrino masses.

1 Introduction

Presently, leaving aside the non-supersymmetric Grand Unified Theories which certainly contradict experiment unless some special extension of the particle spectrum at intermediate scale(s) is made [1], even the commonly accepted $SU(5)$, $SO(10)$ and $E(6)$ SUSY GUTs are far from perfect. The problems, as they appear for the prototype supersymmetric $SU(5)$ model (with a minimal matter, Higgs and gauge boson content) [2], can conventionally be classified as phenomenological and conceptual.

The phenomenological problems include:

- (1) The large value of the strong coupling $\alpha_s(M_Z)$ predicted, $\alpha_s(M_Z) > 0.126$ for the effective SUSY scale $M_{SUSY} < 1$ TeV [3], in contrast to the world average value [4] $\alpha_s(M_Z) = 0.119 \pm 0.002$;
- (2) The predicted proton decay rate, due to colour-triplet $H_c(\bar{H}_c)$ exchange [2], is largely excluded by a combination of the detailed renormalization group (RG) analysis for the gauge couplings [5] and the improved lower limits on the proton decay mode $p \rightarrow \bar{\nu} K^+$ from Super-Kamiokande and on the super-particle masses from LEP2 [6];
- (3) The absence of any sizeable neutrino masses $m_\nu \geq 10^{-2}$ eV in the model is in conflict with the atmospheric neutrino deficit data reported in [7].

Furthermore, the present status of the minimal $SU(5)$ SUSY GUT appears to be inadequate for the following conceptual reasons:

- (4) The first one is of course the doublet-triplet splitting problem for the Weinberg-Salam Higgs field, taken in the fundamental representation of $SU(5)$, which underlies the gauge hierarchy phenomenon in SUSY GUT [2];
- (5) Then a low unification scale M_U is obtained for the MSSM gauge coupling constants, whose value lies one order of magnitude below that of the typical string unification scale $M_{str} \simeq 5.3 \cdot 10^{17}$ GeV [8];
- (6) And lastly, the gravitational smearing of its principal predictions (particularly for $\alpha_s(M_Z)$), due to the uncontrollable high-dimension operators induced by gravity in the kinetic terms of the basic SM gauge bosons [9], makes the ordinary $SU(5)$ model largely untestable.

The abovementioned three plus three problems seem to be generic for all the presently popular GUTs. One could imagine that they are all related, in one way or another, with the still inexplicable gauge hierarchy phenomenon underlying the Grand Unification of quarks and leptons. It even seems possible that the true solution to the doublet-triplet problem would itself specify

the complete framework for Grand Unification, including the fundamental starting gauge symmetry of the GUT involved.

In this connection we would like to call the reader's attention to a novel possibility which opens up [10] in some $SU(N)$ GUTs beyond the prototype $SU(5)$ model: namely, the existence of missing VEV vacuum configurations for an adjoint scalar, which give a natural solution to the doublet-triplet problem. According to this missing VEV mechanism, formulated some time ago [11], the basic symmetry breaking adjoint scalar Σ_j^i ($i, j = 1, \dots, N$) does not develop a VEV in some of the directions in the $SU(N)$ space and, thereby, splits the masses of a pair of Higgs fields H and \overline{H} in a hierarchical way through its coupling with them. The hierarchy is supposed to be such as to give light weak doublets which break the electroweak gauge symmetry and give masses to up and down type quarks, on the one hand, and superheavy colour triplets mediating proton decay, on the other. Interestingly enough, while this possibility is unrealizable in the standard one-adjoint case [11], the situation radically changes when two adjoint scalar fields are used or, equally, when the non-renormalisable higher-order Σ terms are included [10]. As a result, all the above questions appear to be, at least partially, answered. We show in Section 2 that the missing VEV configurations, which ensure the survival of the MSSM at low energies, only emerge in extended $SU(N)$ GUTs with $N \geq 7$. We consider the minimal $SU(7)$ model in detail in the main Section 3 of this paper.

Inasmuch as the missing VEV vacuum must be strictly protected from any large influence coming from the extra symmetry breaking ($SU(7) \rightarrow SU(5)$), we introduce an anomalous $U(1)_A$ symmetry, supposedly inherited from superstrings [12], as a custodial symmetry in the model. The separation of the adjoint scalar and extra symmetry breaking scalar sectors, provided by the $U(1)_A$ symmetry, in fact leads to an increase in the global symmetry of the model. This global symmetry is partially broken, when the $SU(7)$ gauge symmetry is spontaneously broken, thus producing a set of pseudo-Goldstone states of the type

$$5 + \bar{5} + SU(5)\text{-singlets} \tag{1}$$

which gain a mass at the TeV scale due to SUSY breaking. With this exception, which should be considered as the most generic prediction of the model, the spectrum at low energies looks just as if one had the prototype

$SU(5)$ as a starting symmetry. All other $SU(7)$ inherited states, with the chosen assignment of matter and Higgs superfields, acquire GUT scale masses during the symmetry breaking, thus completely decoupling from low-energy physics.

Further, we construct a general R -parity violating superpotential where the effective lepton number violating couplings immediately evolve at the GUT scale, while the baryon number non-conserving ones are safely projected out by the missing VEV vacuum configuration involved. Remarkably, the anomalous $U(1)_A$ introduced purely as a missing VEV protecting symmetry is found to naturally act also as a family symmetry, which can lead to the observed pattern of quark and lepton masses and mixings.

Another distinctive feature of the $SU(7)$ model considered concerns the relatively low mass scale $M_P^{2/3} M_{SUSY}^{1/3}$ [13] of the adjoint moduli fields surviving after $SU(7)$ breaks and, more importantly, their mass-splitting which inevitably appears in the missing VEV generating superpotential. This leads to a very different unification picture in $SU(7)$. String scale gauge coupling unification in the $SU(7)$ model is explicitly demonstrated in Section 4, for both small and large $\tan\beta$ values.

Finally, our conclusions are summarized in Section 5.

2 Motivations for $SU(7)$ GUT

It is well known that a missing VEV solution for an adjoint scalar Σ is absent in the prototype $SU(5)$. Furthermore, even in the general $SU(N)$ GUT, it can not appear in the standard one-adjoint case. The main obstacle to this is the presence of a cubic term Σ^3 in the Higgs superpotential W . This cubic term leads to the impracticable trace condition $Tr\Sigma^2 = 0$ for the missing VEV vacuum configuration, unless there is a special fine-tuned cancellation between $Tr\Sigma^2$ and driving terms stemming from other parts of the superpotential W [11].

On its own the elimination of the Σ^3 term leads to the trivial unbroken symmetry case. However the inclusion of higher even-order Σ terms in the effective superpotential, or the introduction of another adjoint scalar Ω with only renormalizable couplings appearing in W , leads to an all-order missing VEV solution, as was shown in recent papers [10].

Let us consider briefly the high-order term case first. The $SU(N)$ invari-

ant superpotential for an adjoint scalar field conditioned also by the gauge Z_2 reflection symmetry ($\Sigma \rightarrow -\Sigma$)

$$W_A = \frac{1}{2}m\Sigma^2 + \frac{\lambda_1}{4M_P}\Sigma^4 + \frac{\lambda_2}{4M_P}\Sigma^2\Sigma^2 + \dots \quad (2)$$

contains, in general, all possible even-order Σ terms scaled by inverse powers of the (conventionally reduced) Planck mass $M_P = (8\pi G_N)^{-1/2} \simeq 2.4 \cdot 10^{18}$ GeV. As one can readily confirm, the necessary condition for any missing VEV solution to appear in the $SU(N) \otimes Z_2$ invariant superpotential W_A is the tracelessness of all the odd-order Σ terms

$$Tr\Sigma^{2s+1} = 0, \quad s = 0, 1, 2, \dots \quad (3)$$

This condition uniquely leads to a missing VEV pattern of the type

$$\langle \Sigma \rangle = \sigma \text{Diag}(\overbrace{0 \dots 0}^{N-k}, \overbrace{1 \dots 1}^{k/2}, \overbrace{-1 \dots -1}^{k/2}), \quad (4)$$

where the VEV value σ is calculated using the Σ polynomial taken in W_A (2). The vacuum configuration (4) gives rise to a particular breaking channel of the starting $SU(N)$ symmetry

$$SU(N) \rightarrow SU(N-k) \otimes SU(k/2) \otimes SU(k/2) \otimes U(1)_1 \otimes U(1)_2, \quad (5)$$

which we will discuss in some detail a little later. For the moment one may conclude from Eqs. (4, 5) that a missing VEV solution, which retains the ordinary MSSM gauge symmetry $SU(3)_C \otimes SU(2)_W \otimes U(1)_Y$ at low energies, could actually exist if the $SU(5)$ GUT were properly extended.

The superpotential (2) could be viewed as an effective one, following from an ordinary renormalizable two-adjoint superpotential with the second heavy adjoint scalar integrated out. Hereafter, although both approaches are closely related, we deal for simplicity with the two-adjoint case. Towards this end, let us consider in more detail a general $SU(N)$ invariant renormalizable superpotential for two adjoint scalars Σ and Ω , satisfying also the gauge-type Z_2 reflection symmetry ($\Sigma \rightarrow -\Sigma$, $\Omega \rightarrow \Omega$) inherited from superstrings

$$W_A = \frac{1}{2}m\Sigma^2 + \frac{1}{2}M_P\Omega^2 + \frac{1}{2}h\Sigma^2\Omega + \frac{1}{3}\lambda\Omega^3. \quad (6)$$

Here the second adjoint Ω can be considered as a state originating from a massive string mode with the Planck mass M_P . The basic adjoint Σ may be taken at another well motivated scale $m \sim M_P^{2/3} M_{SUSY}^{1/3} \sim O(10^{13})$ GeV [13] where, according to many string models, the adjoint moduli states $(1_c, 1_w)$, $(1_c, 3_w)$ and $(8_c, 1_w)$ (in a self-evident $SU(3)_C \otimes SU(2)_W$ notation) appear. In the present context these states can be identified as just the non-Goldstone remnants Σ_0 , Σ_3 and Σ_8 of the relatively light adjoint Σ which breaks $SU(N)$ in some way. However, all our conclusions remain valid for any reasonable value of m , which is the only mass parameter (apart from M_P) in the model considered. In fact we vary m between 10^{12} and 10^{16} GeV.

As a general analysis of the superpotential W_A (6) shows [10], there are just four possible VEV patterns for the adjoint scalars Σ and Ω : (i) the trivial unbroken symmetry case, $\Sigma = \Omega = 0$; (ii) the single-adjoint condensation, $\Sigma = 0, \Omega \neq 0$; (iii) the "parallel" vacuum configurations, $\Sigma \propto \Omega$ and (iv) the "orthogonal" vacuum configurations, $Tr(\Sigma\Omega) = 0$. The Planck-mass mode Ω , having a cubic term in W_A , in all non-trivial cases develops a standard "single-breaking" VEV pattern

$$\langle \Omega \rangle = \omega \text{Diag}(\overbrace{1 \dots 1}^{N-k}, \overbrace{-\frac{N-k}{k} \dots -\frac{N-k}{k}}^k), \quad (7)$$

which breaks the starting $SU(N)$ symmetry to

$$SU(N) \rightarrow SU(N-k) \otimes SU(k) \otimes U(1). \quad (8)$$

However, in case (iv), the basic adjoint Σ develops the radically new missing VEV vacuum configuration (4), thus giving a "double breaking" of $SU(N)$ to (5). Using the approximation $\frac{h}{\lambda} \gg \frac{m}{M_P}$, which is satisfied for any reasonable values of the couplings h and λ in the generic superpotential W_A (6), the VEV values are given by

$$\omega = \frac{k}{N-k} \frac{m}{h}, \quad \sigma = \left(\frac{2N}{N-k} \right)^{1/2} \sqrt{m M_P} / h \quad (9)$$

respectively. Remarkably, just the light adjoint Σ develops the largest VEV in the model which, for a properly chosen adjoint mass m and coupling constant h , can easily come up to the string scale M_{str} (see Section 4).

Furthermore, as has already been intimated above, in order to have the standard gauge symmetry $SU(3)_C \otimes SU(2)_W \otimes U(1)_Y$ remaining after the breaking (5), one must go beyond the prototype $SU(5)$. As is easily seen from Eqs. (4, 5), there are two principal possibilities: the weak-component and colour-component missing VEV solutions respectively. If it is granted that the "missing VEV subgroup" $SU(N - k)$ in (5) is just the weak symmetry group $SU(2)_W$, as is traditionally argued [11], one is led to $SU(8)$ as the GUT symmetry group ($N - k = 2, k/2 = 3$) [10]. Another, and in fact minimal, possibility is to identify $SU(N - k)$ with the colour symmetry group $SU(3)_C$ in the framework of an $SU(7)$ GUT symmetry ($N - k = 3, k/2 = 2$); this $SU(7)$ model is considered further here. The higher $SU(N)$ GUT solutions, if considered, are based on the same two principal possibilities: the weak-component or colour-component missing VEV vacuum configurations, respectively.

Let us see now how this missing VEV mechanism works to solve the doublet-triplet splitting problem in both $SU(8)$ and $SU(7)$ GUTs. It is supposed that there is a reflection-invariant coupling of the ordinary MSSM Higgs-boson containing supermultiplets H and \overline{H} with the basic adjoint Σ , but not with Ω , in the superpotential W_H :

$$W_H = f_0 \overline{H} \Sigma H \quad (\Sigma \rightarrow -\Sigma, \overline{H} H \rightarrow -\overline{H} H). \quad (10)$$

The superfields H and \overline{H} do not develop VEVs during the first stage of the symmetry breaking. Thus the W_H turns into a mass term for H and \overline{H} determined by the missing VEV pattern (4). This vacuum, while giving generally heavy masses (of the order of the GUT scale M_U) to H and \overline{H} , leaves their weak components strictly massless. To be certain of this, we must specify the multiplet structure of H and \overline{H} for both the weak-component and the colour-component missing VEV vacuum configurations, that is in $SU(8)$ and $SU(7)$ GUTs respectively. In the $SU(8)$ case, H and \overline{H} are fundamental octets whose weak components (ordinary Higgs doublets) do not get masses from the basic coupling (10). In the $SU(7)$ case, H and \overline{H} are 2-index antisymmetric 21-plets which (after projecting out the extra heavy states, see Section 3.3) contain just a pair of massless Higgs doublets. Thus, there certainly is a natural doublet-triplet splitting in both cases and we also have a vanishing μ term at this stage. However, one can readily show that the right order μ term always appears as a result of radiative corrections at the next stage when SUSY breaks [10].

We consider below the minimal $SU(7)$ GUT in some detail.

3 Basics of the $SU(7)$ model

3.1 Matter and Higgs superfields

By analogy with the standard $SU(5)$ model, we consider the simplest anomaly-free combination of the fundamental and antisymmetric 2-index representations of the $SU(7)$ gauge group

$$\left[\bar{\xi}^A + \bar{\zeta}^A + \bar{\Psi}^A + \Psi_{[AB]} \right]_i \quad (11)$$

(where $A, B = 1, \dots, 7$ are $SU(7)$ indices) for each of the three quark-lepton families or generations ($i = 1, 2, 3$). The quark-lepton states reside in the multiplets $\bar{\Psi}^A(\bar{7}) + \Psi_{[AB]}(21)$, while the extra fundamental multiplets $\bar{\xi}^A(\bar{7})$ and $\bar{\zeta}^A(\bar{7})$ are specially introduced in (11) for anomaly cancellation. There is also a set of Higgs superfields, among which are the two already mentioned adjoint Higgs multiplets $\Sigma_B^A(48)$ and $\Omega_B^A(48)$. They are responsible here for the GUT scale breaking of $SU(7)$

$$SU(7) \rightarrow SU(3)_C \otimes SU(2)_W \otimes SU(2)_E \otimes U(1)_1 \otimes U(1)_2 \quad (12)$$

according to the VEVs (4, 7, 9), which now become

$$\langle \Sigma \rangle = \sigma \text{Diag}[0, 0, 0, 1, 1, -1, -1] \quad (13)$$

$$\langle \Omega \rangle = \omega \text{Diag}[1, 1, 1, -\frac{3}{4}, -\frac{3}{4}, -\frac{3}{4}, -\frac{3}{4}] \quad (14)$$

where $\sigma = \sqrt{14mM_P/3}/h$, $\omega = 4m/3h$ and the colour (C), weak (W) and extra (E) components are indicated. In addition there are a pair of Higgs multiplets $H_{[AB]}(21)$ and $\bar{H}^{[AB]}(\bar{21})$ in conjugate representations, which contain the ordinary electroweak doublets. One can easily check that, due to their basic coupling (10) with the adjoint Σ , which develops the missing VEV configuration (13), all the states in the multiplets $H_{[AB]}$ and $\bar{H}^{[AB]}$ become superheavy with mass of order M_U , except for one pair of colour triplets and two pairs of weak doublets. Thus, in order to have just the standard

pair of MSSM electroweak doublets, the other pair of weak doublets should be projected out from the massless state spectrum together with the colour triplet states. This is accomplished by the mixing of $H_{[AB]}$ and $\overline{H}^{[AB]}$ with specially introduced heavy scalar supermultiplets, the 3-index antisymmetric 35-plets of $SU(7)$ $\Phi_{[ABC]}$ and $\overline{\Phi}^{[ABC]}$, which contain just the required states (see Section 3.3). And, finally, there are two fundamental scalar superfields $\varphi_A(7)$ and $\eta_A(7)$ and their "conjugates" $\overline{\varphi}^A(\overline{7})$ and $\overline{\eta}^A(\overline{7})$ which break the extra symmetry at the GUT scale. We consider this key question first.

3.2 Extra symmetry breaking

Inasmuch as the extra symmetry should also be broken

$$SU(7) \rightarrow SU(5) \tag{15}$$

at the GUT scale, in order not to spoil gauge coupling unification, a question arises: how can the adjoint Σ missing VEV configuration (13) survive so as to be subjected to at most a shift of order the electroweak scale? This requires, in general, that the superpotential W_A (6) be strictly protected from any large influence of the scalars φ and η , which provide the extra symmetry breaking (15). Technically, such a custodial symmetry may be a superstring-inherited anomalous $U(1)_A$ [12], which induces a high-scale extra symmetry breaking (15) through the Fayet-Iliopoulos (FI) D -term [14]:

$$D_A = \xi + \sum Q_A^n |\langle \mathcal{F}^n \rangle|^2, \quad \xi = \frac{Tr Q_A}{192\pi^2} g_{str}^2 M_P^2. \tag{16}$$

Here the sum runs over all "charged" scalar fields in the theory, including those which do not develop VEVs and only contribute to $Tr Q_A$. For realistic or semi-realistic models, $Tr Q_A$ has turned out to be quite large, $Tr Q_A = O(100)$ (see the recent discussion in [15]). So, the spontaneous breaking scale of the $U(1)_A$ symmetry and the related extra symmetry (15) is naturally located at the string scale. The protecting anomalous $U(1)_A$ symmetry is supposed to keep the scalars φ and η essentially decoupled from the basic adjoint superpotential W_A (6), so as not to strongly influence its missing VEV vacuum configuration (13) through the appearance of potentially dangerous couplings of the type $\overline{\varphi}\Sigma\varphi$ and $\overline{\eta}\Sigma\eta$.

Anyway, once a separation of the adjoint scalar and extra symmetry breaking scalar sectors takes place in the supersymmetric $SU(7) \otimes U(1)_A$

theory considered, an accidental global symmetry $SU(7)_{\Sigma-\Omega} \otimes U(7)_{\varphi-\eta}$ appears. This global symmetry is radiatively broken and one or two families of pseudo-Goldstone (PG) states of type (1) are produced at a TeV scale, where SUSY softly breaks [10]. The two-family case corresponds to the most degenerate Higgs potential, where the scalars φ and η are only allowed to appear through the basic $SU(7)$ and $U(1)_A$ D -terms and, thereby, increase their global symmetry to $U(7)_{\varphi} \otimes U(7)_{\eta}$. This two-family case occurs when the $U(1)_A$ charges of the bilinears $\overline{\varphi}\varphi$, $\overline{\eta}\eta$, $\overline{\varphi}\eta$ and $\overline{\eta}\varphi$ are all positive (or all negative), so that they can not appear in the $SU(7) \otimes U(1)_A$ invariant superpotential in any order.

However, remarkably, it is possible for the adjoint and fundamental scalar sectors in the superpotential to overlap without disturbing the adjoint missing VEV configuration. This naturally occurs when the scalars φ and η are conditioned by the $U(1)_A$ symmetry to develop orthogonal VEVs along the "extra" directions

$$\varphi_A = \delta_{A6} V_1, \quad \eta_A = \delta_{A7} V_2 \quad (17)$$

The simplest choice of such safe mixing terms is given by dimension-5 operators, invariant under the reflection symmetry $\Sigma \rightarrow -\Sigma$, $\overline{\varphi} \rightarrow -\overline{\varphi}$, $\overline{\eta} \rightarrow -\overline{\eta}$, of the type¹

$$W_{H1} = \frac{1}{M_P} \overline{\eta} [a \cdot S \Sigma + b \cdot \overline{\varphi} \varphi] \varphi \quad (18)$$

The dimensionless coupling constants a and b are both of order $O(1)$ and S is some new singlet superfield which gets its VEV through the FI D -term (16), just as the scalars φ and η do. One can consider the field S as a basic carrier of unit $U(1)_A$ charge in the model. In terms of its charge, the charges of the bilinears in W_{H1} are determined to be +1 for $\overline{\varphi}\varphi$ and -1 for $\overline{\eta}\varphi$, while the charges of the bilinears $\overline{\eta}\eta$ and $\overline{\varphi}\eta$ are not yet determined from the couplings (18). The latter charges can be taken to be positive, as follows when we consider the other coupling terms in Sections 3.3 - 3.6. This implies that any terms containing φ and η scalars in the superpotential must also include the bilinear $\overline{\eta}\varphi$, so as to properly compensate the $U(1)_A$ charges. However, for a vacuum configuration where the orthogonality condition $\overline{\eta}\varphi = 0$ naturally arises, this gives an all-order solution excluding the dangerous $\overline{\varphi}\Sigma\varphi$ and $\overline{\eta}\Sigma\eta$

¹Enlarging the scalar sector properly one can write a renormalizable superpotential as well.

terms. In fact this orthogonality condition is precisely one of the conditions satisfied at the SUSY invariant global minimum of the Higgs potential, as follows from the vanishing F -terms of the superfields involved in (18):

$$\bar{\eta}\varphi = 0, \quad \bar{\varphi}\varphi = \frac{a}{b} \cdot S\sigma \quad (19)$$

Here the VEV value, appearing on the extra symmetry components of the adjoint Σ (13), has been used. One can now readily see that a non-diagonal mass-term appears for the PG states related with the multiplets φ and η and their "conjugates"

$$M_{\bar{\eta}\varphi} \equiv [W''_{H1}]_{\bar{\eta}\varphi} = \frac{a}{M_P} \cdot S(\Sigma + \sigma I) \quad (20)$$

where I is the 7×7 unit matrix. This explicitly shows that one superposition of the two PG $5 + \bar{5}$ families (1) becomes heavy, while the other is always left massless. In fact this result is a general consequence of the symmetry breaking pattern involved. The point is that neither of the other mass-terms $M_{\bar{\varphi}\eta}$, $M_{\bar{\varphi}\varphi}$ and $M_{\bar{\eta}\eta}$ can be allowed by the $U(1)_A$ symmetry for any generalization of the superpotential W_{H1} (18); otherwise the dangerous $\bar{\varphi}\Sigma\varphi$ and $\bar{\eta}\Sigma\eta$ couplings inevitably appear as well. Similarly, in the general $SU(N)$ case, it can be shown [16] that at least one family of PG states of type (1) always exists. Together with the ordinary quarks and leptons and their superpartners these PG states, both bosons and fermions, determine the particle spectrum at low energies. In most of what follows the existence of just one family of PG states at the sub-TeV scale will be assumed.

3.3 Heavy states

We demonstrate below that, when the extra symmetry is broken (15), all the states in the $SU(7)$ model, beyond the ordinary MSSM particle spectrum plus one family of pseudogoldstones (1), acquire masses of order the GUT scale. An exception can be made for the sterile states (the states in the matter multiplets (11) having the extra symmetry charges only) whose fermionic components might be referred to as sterile neutrinos (see Section 3.4).

First of all let us consider the Higgs sector and show that all the states in the basic Higgs multiplets $H_{[AB]}$ and $\bar{H}^{[AB]}$ become superheavy, except

for one pair of weak doublets. One can readily check that, when the colour-component missing VEV solution (13) is substituted into the superpotential W_H (10), superheavy masses are generated for most of the components of the H and \overline{H} multiplets. However, the following states (weak, colour and extra symmetry components are explicitly indicated)

$$H_{w6} , \quad \overline{H}^{w6} , \quad H_{w7} , \quad \overline{H}^{w7} , \quad H_{[cc']} , \quad \overline{H}^{[cc']} \quad (21)$$

remain massless at this stage of $SU(7)$ symmetry breaking. Therefore one of the two pairs of weak doublets in (21), as well as the colour triplets, must also become heavy in order to obtain the ordinary picture of MSSM at low energies. This happens as a result of mixing H and \overline{H} with the specially introduced (see Section 3.1) superheavy scalar supermultiplets $\Phi_{[ABC]}$ and $\overline{\Phi}^{[ABC]}$ in the basic Higgs superpotential

$$W_{H2} = f \cdot H \overline{\Phi} \varphi + \overline{f} \cdot \overline{H} \Phi \overline{\varphi} + y \cdot S \overline{\Phi} \Phi. \quad (22)$$

Here f , \overline{f} and y are dimensionless coupling constants. When the scalars φ and η get their VEVs, thus breaking the extra symmetry, the required mixing terms are generated. It is worth noting that the presence of the "conjugated" $\overline{\Phi} - H$ and $\Phi - \overline{H}$ mixings in W_{H2} could allow the dangerous $\overline{\varphi} \Sigma \varphi$ and $\overline{\eta} \Sigma \eta$ terms, destroying the missing VEV solution, unless the bilinear term $\overline{\Phi} \Phi$ has a nonzero $U(1)_A$ charge. Therefore, this term appears in W_{H2} together with the singlet scalar superfield S – the basic $U(1)_A$ charge carrier introduced earlier in W_{H1} (18).

It is easy to see now that the W_{H2} couplings (22) will rearrange the mass spectrum of the states (21), so as to leave just a standard pair of MSSM electroweak doublets massless. Considering the mixing of the colour triplet states first, one can see from the 2×2 mass matrix for the states $H_{[cc']}$ and $\overline{H}^{[cc']}$ and the double-coloured components $\Phi_{[cc'6]}$ and $\overline{\Phi}^{[cc'6]}$ that, when properly diagonalized, the colour components in (21) obtain a mass M_* of order

$$M_* \sim \frac{f \overline{f}}{y} \frac{\langle \varphi \rangle \langle \overline{\varphi} \rangle}{S} \sim M_U. \quad (23)$$

The combination of primary coupling constants f , \overline{f} and y in (23), can be taken $O(1)$ in general. For the weak doublet case, there is a 3×3 mass matrix corresponding to the mixing of the states H_{w6} , H_{w7} and $\Phi_{[w67]}$, and

their "conjugates" \overline{H}^{w6} , \overline{H}^{w7} and $\overline{\Phi}^{[w67]}$ respectively. After diagonalization this matrix leaves, as can readily be checked, just one pair of weak-doublets H_{w6} and \overline{H}^{w6} strictly massless, while the other pair H_{w7} and \overline{H}^{w7} acquires a mass of order M_* (23).

It seems reasonable to assume that the components of H and \overline{H} , which get masses from their direct coupling (10) with the basic adjoint Σ , should have a mass of order M_U , while the states in (21), which develop a mass M_* from their mixing with the heavy Φ and $\overline{\Phi}$ multiplets (22), could naturally be relatively light, $M_* = O(10^{-2} \div 1)M_U$. In this case, the proton decay inducing colour-triplet states H_{c6} and \overline{H}^{c6} , which are partners of the light weak-doublets H_{w6} and \overline{H}^{w6} , are located at the GUT scale M_U , while the double-coloured states $H_{[cc']}$ and $\overline{H}^{[cc']}$ (21), which can not induce proton decay, are relatively light. Thus the problem of an unacceptably fast proton decay, due to dimension-5 operators, would be naturally solved in the $SU(7)$ model considered. At the same time the double-coloured states $H_{[cc']}$ and $\overline{H}^{[cc']}$, if taken relatively light, would properly contribute to the running of the gauge coupling constants (see Section 4).

Next we consider the additional matter multiplets and, two anti-septets $\overline{\xi}^A$ and $\overline{\zeta}^A$ for each of the three generations, which were introduced (Section 3.1) for $SU(7)$ anomaly cancellation. They are assumed to form their masses by combining with the basic 2-index multiplet $\Psi_{[AB]}$ of their own generation

$$W_{Y1} = g_\xi \cdot \overline{\xi}^A \Psi_{[A6]} \overline{\varphi}^6 + g_\zeta \cdot \overline{\zeta}^A \Psi_{[A7]} \overline{\eta}^7. \quad (24)$$

Here the $SU(7)$ index A and the extra symmetry subgroup indices 6 and 7, along which the VEVs (17) develop, are explicitly indicated (g_ξ, g_ζ are coupling constants). So, again one can see that any state in the multiplets $\overline{\xi}$ and $\overline{\zeta}$ (for all three generations) carrying colour and/or electric charge acquires a mass of order M_U .

3.4 Sterile neutrinos

The states in the matter multiplets (11) of each generation, which only have charges under the extra symmetry

$$\Psi_{[67]}, \quad \overline{\Psi}^6, \quad \overline{\Psi}^7, \quad \overline{\xi}^6, \quad \overline{\xi}^7, \quad \overline{\zeta}^6, \quad \overline{\zeta}^7 \quad (25)$$

are of particular interest as possible $SU(7)$ candidates for sterile neutrinos. One can see that two superpositions of $\Psi_{[67]}$, $\bar{\xi}^7$ and $\bar{\zeta}^6$ acquire masses of order $\langle \bar{\varphi} \rangle \sim \langle \bar{\eta} \rangle \sim M_U$ from the couplings (24). As to the other states in (25), their masses depend on the $U(1)_A$ charges assigned to the matter and Higgs superfields involved. If one takes the simple set of charges presented in Table 1, all of them in fact can get masses from the allowed high-order terms (inherited from superstrings or induced by gravitational corrections) of the type

$$\begin{aligned}
M_P W_{Y_2} = & [a_1 \bar{\Psi} \bar{\Psi} + a_2 \bar{\Psi} \bar{\zeta} \frac{S}{M_P} + a_3 \bar{\zeta} \bar{\zeta} (\frac{S}{M_P})^2 + a_4 \bar{\Psi} \bar{\xi} (\frac{S}{M_P})^3] \cdot \varphi \eta + \\
& [a_5 \bar{\zeta} \bar{\xi} + a_6 \bar{\xi} \bar{\xi} (\frac{S}{M_P})^2] \cdot \eta \eta + \dots
\end{aligned} \tag{26}$$

where the higher order terms indicated by dots are ignored (overall $SU(7)$ index contraction is implied and the dimensionless coupling constants a_1, \dots, a_6 are assumed to be all of $O(1)$). It follows, from the couplings in W_{Y_1} (24) and W_{Y_2} (26), that for each generation the symmetric 7×7 mass matrix of the sterile states (25) has all its eigenvalues non-zero in general. The smallest eigenvalue is just given by the order of the highest term kept in the superpotential W_{Y_2} and is thus of order $\frac{M_U^5}{M_P^4}$.

Some of these sterile states should be considered as candidates for right-handed neutrinos. The traditionally used Planck or GUT mass scale right-handed neutrino leads, via the well-known see-saw mechanism [19], to ordinary neutrino masses in the range $m_\nu = 10^{-5} \div 10^{-3}$ eV or lower, which cannot explain the recent SuperKamiokande atmospheric neutrino data [7]. By contrast, the sterile states discussed here² could naturally lead to neutrino mass(es) in just the required region $m_\nu = 10^{-2} \div 1$ eV.

Interestingly, with the choice of $U(1)_A$ charges taken in Table 1, the direct Higgs-matter mixing terms such as

$$W_{Y_3} = b_1 \bar{\xi} \eta S + b_2 \bar{\Psi} \varphi \frac{S^2}{M_P} + b_3 \bar{\zeta} \varphi \frac{S^3}{M_P^2} + \dots \tag{27}$$

²Other potentially interesting right-handed neutrino candidates, also well motivated in the $SU(7)$ model, are the fermionic superpartners of the adjoint moduli states Σ_0 and Σ_3 at a scale of $M_P^{2/3} M_{SUSY}^{1/3}$ stemming from superstrings [13].

can also evolve, provided that R -parity symmetry is not assumed. They will induce the condensation of some of the heavy sterile sneutrinos, just like takes place for ordinary sneutrinos in the bilinear Higgs-lepton mixing model [18].

At the same time, it is well to bear in mind that some other choice of the $U(1)_A$ charges, different from those in Table 1, could (partially or completely) forbid the couplings (26) and (27), thus leading to certain light and even strictly massless sterile neutrinos. As is well known, they are not ruled out by experiment [7],[18].

3.5 Lepton number violation

One would think that there is no fundamental reason for exact R -parity (RP) symmetry in the framework of supersymmetric GUTs, where not only fermions but also their scalar superpartners are the natural carriers of lepton and baryon numbers. Accordingly, we suppose that all the generalized Yukawa couplings, the RP -conserving (ordinary up and down fermion Yukawas), as well as the RP -violating ones allowed by $SU(7) \otimes U(1)_A$ symmetry, are given by a similar set of dimension-5 operators. We note that the usual dimension-4 trilinear Yukawa couplings are forbidden by the underlying gauge invariance. The dimension-5 operators take the form ($i, j, k = 1, 2, 3$ are the generation indices)

$$\mathcal{O}_{ij}^{up} = \frac{G_{ij}^u}{M_P} (\Psi_i \Psi_j) (H \eta) \quad (28)$$

$$\mathcal{O}_{ij}^{down} = \frac{G_{ij}^d}{M_P} (\bar{\Psi}_i \Psi_j) (\bar{H} \varphi) \quad (29)$$

$$\mathcal{O}_{ijk}^{rpv} = \frac{G_{ijk}}{M_P} (\bar{\Psi}_i \Psi_j) (\bar{\Psi}_k \Sigma) \quad (30)$$

and are collected in the Yukawa part of the superpotential W_Y . As specified above (Section 3.1), each of the three generations of quarks and leptons lies in two multiplets $\bar{\Psi}^A + \Psi_{[AB]}$ of $SU(7)$. Also the ordinary electroweak doublets are contained in the multiplets $H_{[AB]}$ and $\bar{H}^{[AB]}$, while the scalars φ and η and their "conjugates", which break $SU(7)$ to $SU(5)$, are fundamental septets and anti-septets, respectively.

Now, on substituting VEVs for the scalars Σ (13), φ and η (17) in the basic operators (28–30), one obtains at low energies the effective renormalisable Yukawa and lepton number violating (LNV) interactions with coupling constants

$$Y_{ij}^u = G_{ij}^u \frac{\langle \eta \rangle}{M_P} , \quad Y_{ij}^d = G_{ij}^d \frac{\langle \varphi \rangle}{M_P} , \quad \Lambda_{ijk} = G_{ijk} \frac{\langle \Sigma \rangle}{M_P} . \quad (31)$$

At the same time the baryon number violating (BNV) couplings prove to be completely eliminated. The key element here turns out to be that the adjoint field Σ , involved in the effective couplings (30), develops a VEV configuration with strictly zero colour components (13) in the SUSY limit. However, at the next stage when SUSY breaks, the radiative corrections will shift the missing VEV components of Σ to some nonzero values of order M_{SUSY} . In this way the ordinary μ -term of the MSSM, on the one hand, and baryon number violating couplings with hierarchically small coupling constants of the order M_{SUSY}/M_U , on the other, are induced. So, our missing VEV solution to the gauge hierarchy problem leads in fact to a similar hierarchy of baryon vs lepton number violation.

The \mathcal{O} -operators (28–30) can be viewed as effective interactions generated through the exchange of some superheavy states, which we can interpret as massive string modes. When they are generated by the exchange of the same superheavy multiplet (formed from a pair of fundamental septets $7 + \bar{7}$), the operators (29) and (30) appear with the dimensionless effective coupling constants aligned in flavour space [17]:

$$\Lambda_{ijk} = Y_{ij}^d \cdot \epsilon_k . \quad (32)$$

Here the parameters ϵ_k ($k = 1, 2, 3$) include some known combination of the primary dimensionless coupling constants and a ratio of the VEVs of the scalars Σ and φ . This strict relation, between general RP -violating and down fermion Yukawa coupling constants, is then split into the ones for charged lepton (cl) and down quark (dq) LNV couplings respectively,

$$\lambda_{ijk} = Y_{ij}^{cl} \cdot \epsilon_k , \quad \lambda'_{ijk} = Y_{ij}^{dq} \cdot \epsilon_k , \quad (33)$$

when it is evolved from the GUT scale down to low energies.

Therefore, the postulated common origin of all the generalized Yukawa couplings, both RP -conserving and RP -violating, at the GUT scale results in some minimal form of lepton number violation, with the proviso that appropriate mediating superheavy-states exist. As a result, all significant physical manifestations of LNV reduce to those of the effective trilinear couplings³ $LL\bar{E}$ and $LQ\bar{D}$ aligned, both in magnitude and orientation in flavour space, with the down fermion (charged lepton and down quark) effective Yukawa couplings. However the effective bilinear terms $\mu_i L_i H$ appear to be generically suppressed by the custodial $U(1)_A$ symmetry involved. Detailed phenomenology of this model related to the flavor-changing processes both in quark and lepton sectors, radiatively induced neutrino masses and decays of the LSP can be found in a recent paper [17].

3.6 Masses and mixings of quarks and leptons

So far we have used an anomalous $U(1)_A$ symmetry purely as a missing VEV protecting symmetry in the $SU(7)$ model. In Table 1 we list the $U(1)_A$ ($e^{iQ_A\theta}$) and Z_2 ($e^{inz\pi}$) charges, which allow just the couplings appearing in the total superpotential W_T :

$$W_T = W_A + W_H + W_{H1} + W_{H2} + W_Y + W_{Y1} + W_{Y2} + W_{Y3} \quad (34)$$

(see Eqs. (6, 10, 18, 22, 24, 26-30)). The superpotential W_T in turn completely fixes all the Q_A charges in terms of the charge of the singlet scalar superfield S , which we take to be unity $Q_A^S = 1$. Remarkably, as one can see from Table 1, the Q_A charges of the scalar bilinears $\bar{\varphi}\varphi$ and $\bar{\eta}\eta$, which break the extra symmetry ($SU(7) \rightarrow SU(5)$), happen to be positive, while the adjoint scalar Σ has $Q_A = 0$. Hence the potentially dangerous $\bar{\varphi}\Sigma\varphi$ and $\bar{\eta}\Sigma\eta$ couplings are forbidden, thus providing an all-order solution to the doublet-triplet splitting problem in the $SU(7)$ model.

However, together with its protecting function, it is tempting to treat the anomalous $U(1)_A$ also as a family symmetry [20]. Then the zero charges Q_A presented in Table 1 for the matter multiplets $\Psi_{[AB]}$ and $\bar{\Psi}^A$ would only be assigned to the third family of quarks and leptons, while the other two families would be assigned some nonzero Q_A charges. The observed hierarchy of

³Here L and Q denote lepton and quark $SU(2)$ doublet superfields, while \bar{E} and \bar{D} denote lepton and down quark $SU(2)$ singlet superfields.

quark-lepton masses and mixings might then be generated via the Froggatt-Nielsen mechanism [21]. In the case at hand this hierarchy is described by the natural expansion parameter $\epsilon = \frac{\langle S \rangle}{M_P} \sim \frac{M_{str}}{M_P} (\simeq 0.2)$. One can readily see that a simple choice of Q_A charges for the matter multiplets $\Psi_{[AB]i}$ and $\bar{\Psi}_i^A$ (as explicitly indicated in brackets in units of Q_A^S) leads to Yukawa matrices of the type

$$Y^u \propto \begin{matrix} & \Psi_1^{(-3)} & \Psi_2^{(-2)} & \Psi_3^{(0)} \\ \begin{matrix} \Psi_1^{(-3)} \\ \Psi_2^{(-2)} \\ \Psi_3^{(0)} \end{matrix} & \begin{pmatrix} \epsilon^6 & \epsilon^5 & \epsilon^3 \\ \epsilon^5 & \epsilon^4 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} \end{matrix} \quad (35)$$

and

$$Y^d \propto \begin{matrix} & \Psi_1^{(-3)} & \Psi_2^{(-2)} & \Psi_3^{(0)} \\ \begin{matrix} \bar{\Psi}_1^{(-2)} \\ \bar{\Psi}_2^{(0)} \\ \bar{\Psi}_3^{(0)} \end{matrix} & \begin{pmatrix} \epsilon^5 & \epsilon^4 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} \end{matrix}. \quad (36)$$

These matrices yield a quark and charged lepton mass hierarchy similar to the observed pattern, as well as the right orders of magnitude for the CKM matrix elements (see also [22]). Phenomenologically, in the case of Y^u (35) the constant of proportionality is of order unity. However, in the case of Y^d (36), the constant of proportionality is only of order unity for large values of $\tan \beta$. In the case of small $\tan \beta$ it is necessary to assume that the underlying fundamental couplings give a constant of proportionality of order 0.01^4 .

There is another important aspect of the model related with masses of quarks and leptons which is worthy of special note. The $SU(7)$ GUT considered, as well as the prototype $SU(5)$ model, predicts (see Eq.(29)) the equality of the down quark and charged lepton masses at the grand unification scale. However, due to the string scale unification in the $SU(7)$ model,

⁴It must be admitted that the origin of this small factor is not understood, just as the origin of a large value of $\tan \beta$ is not understood. Nonetheless, it is possible to generate this hierarchical bottom to top quark mass ratio, by uniformly subtracting two units of $U(1)_A$ charge from all down fermion multiplets $\bar{\Psi}_1$, $\bar{\Psi}_2$ and $\bar{\Psi}_3$. The Y^d matrix is then multiplied by an overall factor of order $O(\epsilon^2)$. However, we do not make use of this option here.

the influence of the gravitational corrections on the Yukawa couplings is much more important than in the prototype $SU(5)$ [25]. Actually, in general, one must include $SU(7) \otimes U(1)_A \otimes Z_2$ invariant corrections to the down fermion Yukawa couplings (29, 31, 36) of the type

$$\mathcal{O}_{ij, cor}^{down} \sim \frac{\epsilon^{|\bar{q}_i + q_j|}}{M_P^3} (\bar{\Psi}_i \Sigma^2 \Psi_j) (\bar{H} \varphi) \quad (37)$$

where $\bar{q}_i = Q_A(\bar{\Psi}_i)$ and $q_j = Q_A(\Psi_j)$ are the $U(1)_A$ charges for the matter multiplets given in Eq. (36). These gravitational corrections break the equality between the down quark and charged lepton Yukawa couplings at the unification scale. They give rise to effective dimensionless coupling constants Y'_{ij} of order $\frac{\langle \Sigma \rangle^2}{M_P^2}$ times the right hand side of Eq. (36). Thus, taking $\langle \Sigma \rangle$ to be of order the string scale, the corrections Y'_{ij} turn out to be of the same order as the physical Yukawa couplings Y_{ij}^d (e.g. $Y'_{33} \sim Y_{b,\tau} \sim 0.01$) in the small $\tan \beta$ case. Thereby, under the circumstances considered (string-scale unification plus small $\tan \beta$), gravitational corrections are expected not only to spoil the $SU(5)$ quark-lepton mass predictions for the d and s quarks, as argued in [25], but also for the b quark. At the same time these corrections will not significantly disturb the hierarchical structure of the mass matrices (35) and (36), which is essentially protected by the $U(1)_A$ gauge symmetry. In the large $\tan \beta$ case the gravitational corrections are quite small even for a string-scale unification and so the $b - \tau$ Yukawa unification, properly corrected by supersymmetric loop contributions (see further discussion in Section 4), is actually predicted as in the $SU(5)$ model.

4 Gauge coupling unification

We now consider gauge coupling unification in the above $SU(7)$ model. At the low-energy scale, in addition to the three standard families of quarks and leptons (and squarks and sleptons), there is just one family of PG bosons (1) and their superpartners which will modify the running of the gauge and Yukawa couplings in the model. However, as we will see, the contribution of the split adjoint moduli Σ_3 and Σ_8 taken at their natural scale $M_P^{2/3} M_{SUSY}^{1/3}$ (for earlier works see [13], [23]) turns out to be much more important.

We start with the reminder that, to one-loop order, gauge coupling unification is given by the three renormalization group (RG) equations, relating

the values of the gauge couplings at the Z-peak $\alpha_i(M_Z)$ ($i = 1, 2, 3$) and the common gauge coupling α_U at the unification scale M_U [1]:

$$\alpha_i^{-1} = \alpha_U^{-1} + \sum_p \frac{b_i^p}{2\pi} \ln \frac{M_U}{M_p}. \quad (38)$$

Here b_i^p are the three beta function coefficients, corresponding to the $SU(7)$ subgroups $U(1)_Y$, $SU(2)_W$ and $SU(3)_C$ respectively, for the particle labeled by p . The sum extends over all the contributing particles in the model, and M_p is the mass threshold at which each decouples. All of the SM particles, and also the second electroweak doublet of MSSM, are taken to be present at the starting scale M_Z . The next contribution enters at the supersymmetric threshold, associated with the decoupling of the supersymmetric particles at some single effective scale M_{SUSY} [3]. We propose to use relatively low values for this scale, $M_{SUSY} \sim M_Z$, so as to keep sparticle masses typically in the few hundred GeV region. Furthermore, the PG states (1) are also taken at a sub-TeV scale. As to the heavy states, there are basic thresholds relating to the adjoint moduli Σ_8 and Σ_3 , with masses M_8 and M_3 respectively, and the thresholds caused by certain components in the $SU(7)$ Higgs multiplets $H_{[AB]}$ and $\bar{H}^{[AB]}$,

$$H_{w7}, \bar{H}^{w7}, H_{[cc']}, \bar{H}^{[cc']}, \quad (39)$$

developing masses of order M_* (23), which we argued in Section 3.3 could lie somewhat lower than the GUT scale M_U . We refer to the latter as the H -states. All other states in the Higgs, matter and gauge multiplets, including the superheavy gauge bosons and their superpartners, do not contribute to Eq. (38), for they are assumed to lie at the GUT scale M_U , above which all particles fill complete $SU(7)$ multiplets.

Now, the "matching" equation for the gauge couplings (38) reads as follows:

$$12\alpha_2^{-1} - 7\alpha_3^{-1} - 5\alpha_1^{-1} = \frac{3}{2\pi} \left(-2 \ln \frac{M_U}{M_*} + \ln \frac{M_U}{M_3} - 7 \ln \frac{M_3}{M_8} - \frac{19}{6} \ln \frac{M_{SUSY}}{M_Z} \right) \quad (40)$$

This can be viewed as the basis for understanding qualitatively the constraint on the value of $\alpha_3 = \alpha_s(M_Z)$ from grand unification, and its dependence on

the present precision electroweak measurements of α_1 and α_2 [4] and all the thresholds involved. One can see from Eq. (40) that $\alpha_s(M_Z)$ increases with $\frac{M_U}{M_3}$ and decreases with $\frac{M_U}{M_*}$, $\frac{M_{SUSY}}{M_Z}$ and, especially, with $\frac{M_3}{M_8}$ (the term with the largest coefficient in front of the logarithm). Paradoxically enough, in the absence of all threshold effects ($M_* = M_3 = M_8 = M_U$ and $M_{SUSY} = M_Z$), Eq. (40) leads to a phenomenologically acceptable value of $\alpha_s(M_Z)$

$$\alpha_s = \frac{7\alpha_{em}}{15\sin^2\theta_W - 3} = 0.117, \quad (41)$$

using the values $\alpha_{em}^{-1} = 127.9$ and $\sin^2\theta_W = 0.2313$ taken at M_Z [4]. Unfortunately, this value increases unacceptably when 2-loop order corrections are included. In the standard $SU(5)$ case [2], with degenerate adjoint moduli Σ_3 and Σ_8 ($M_3 = M_8$ at the GUT scale), an unacceptably high value of α_s is obtained even if all the possible threshold effects are taken into account [3, 5, 6].

However, a drastically different unification picture appears when a generic mass splitting between Σ_3 and Σ_8 , which is a consequence of the missing VEV inducing superpotential (6), is taken into account⁵. Actually, after the adjoint scalars Σ and Ω develop their VEVs (13), the physical masses of the surviving adjoint moduli Σ_3 and Σ_8 turn out to be fixed. Diagonalization of the common Σ - Ω mass matrix

$$M_{ab} = [W''_{ab}]_{<\Sigma>, <\Omega>} \quad (42)$$

(where the indices a and b stand for the corresponding components of Σ and Ω) leads to

$$M_3 = \frac{14m}{3}, \quad M_8 = \frac{7m}{3}, \quad \frac{M_3}{M_8} = 2 \quad (43)$$

in contrast to $M_3/M_8 = 1$ in the standard $SU(5)$ model [2].

So, with the above observations, we are now ready to carry out the standard two-loop analysis (with conversion from the \overline{MS} scheme to the \overline{DR} one

⁵This two-adjoint superpotential was recently applied to the $SU(5)$ model [23]. Although there can not be a missing VEV solution in the framework of $SU(5)$, a generic mass-splitting between the adjoint moduli Σ_3 and Σ_8 appears ($\frac{M_3}{M_8} = 4$) that leads to a natural string-scale unification without any extension of the matter or Higgs sectors in the model.

included) [1, 24] for the gauge ($\alpha_1, \alpha_2, \alpha_3$) and Yukawa (α_t, α_b and α_τ) coupling evolution. Here we are using the notation $\frac{Y_{t,b,\tau}^2}{4\pi} \equiv \alpha_{t,b,\tau}$ for the top- and bottom-quarks and the tau-lepton. We include the standard supersymmetric threshold corrections at low energies, taken at a single scale $M_{SUSY} = M_Z$, and those related with the PG states (1) taken also at a sub-TeV scale, namely at 300 GeV. The heavy threshold corrections due to the H states (39) with mass M_* are taken close to the grand unification scale M_U . As to the heavy Σ adjoint moduli, their masses were treated differently in the two parts of our calculation. When making predictions of $\alpha_s(M_Z)$ as a function of M_U (see Figure 1), they were varied from the MSSM unification point $M_U^0 \simeq 2 \cdot 10^{16}$ GeV ($M_3 = M_U^0$, $M_8 = \frac{1}{2}M_U^0$) down to the intermediate value $m = O(10^{13})$ GeV [13], thus pushing M_U up to the string scale M_{str} . However, for the study of the string-scale unification case $M_U = M_{str}$ (see Tables 2 and 3), these masses can be predicted and they, in fact, turned out to be at their natural scale $M_P^{2/3} M_{SUSY}^{1/3}$ stemming from superstrings [13]. The mass splitting between the weak triplet Σ_3 and the colour octet Σ_8 , which is fixed at the unification scale M_U according to (43), noticeably decreases as M_3 and M_8 run down from M_U to the lower energies according to their own two-loop RG evolution. This effect was included in the analysis.

As to the Yukawa coupling evolution, we have considered both the cases of small and large values of $\tan \beta$. The first case corresponds to α_t having a large enough value at the unification scale M_U ($0.1 > \alpha_t(M_U) > 0.01$) that it evolves towards its infrared fixed point, while $\alpha_b(M_U)$ and $\alpha_\tau(M_U)$ are significantly smaller ($\alpha_{b,\tau}(M_U) \lesssim 10^{-4}$). By requiring the RG evolved value of $\alpha_t(m_t)$ to reproduce the observed value of the top quark pole mass, $m_t = 175 \pm 6$ GeV, the values of $\tan \beta$ in Table 2 were determined. These values naturally satisfy the usual RG infrared fixed point bound $\tan \beta > 1.5$, which is just consistent with the present experimental lower limit on the lightest MSSM Higgs mass [4].

At the same time the direct $b-\tau$ unification prediction at the string scale, $R_{b\tau}(M_U) = 1$, for the bottom quark to tau lepton mass ratio does not work well, when evolved down to low energies (see Table 2) and compared to the experimental value [4]

$$R_{b\tau}^{\text{exp}}(M_Z) = 1.6 \pm 0.2 \quad (44)$$

However, as we argued in Section 3.6, the $SU(5)$ -like equalities of down

quark and charged lepton masses, including $m_b(M_U) = m_\tau(M_U)$, are highly unstable under gravity corrections for small $\tan\beta$ in the string-scale $SU(7)$ GUT. So they are not expected to work well in our model.

The case of large $\tan\beta$, where all the couplings $\alpha_{t,b,\tau}$ are relatively large and can approach a fixed-point, was found to have acceptable solutions over the whole of the following range of starting values: $0.01 < \alpha_t(M_U) < 0.03$ and $0.001 < \alpha_{b,\tau}(M_U) < 0.3$. Here, however, the predictions for the top quark pole mass m_t , the ratio $R_{b\tau}(M_Z)$ and $\tan\beta$ depend on detailed information about the superparticle mass spectrum. Generally, in the large $\tan\beta$ case, large supersymmetric loop contributions to the bottom quark mass m_b are expected, which make the top mass prediction uncertain as well. Since, fortunately, the direct SUSY loop contributions to the t quark and τ lepton masses are quite small [26], one can adopt the following calculational strategy: calculate the value of $\tan\beta$ using $\alpha_\tau(m_\tau)$ from the RG equations and the observed tau lepton mass $m_\tau = 1.777$ GeV and use this value of $\tan\beta$ to obtain the m_t value. Further, for the given top mass m_t one can calculate the size of the SUSY loop corrections to the bottom mass δm_b required to bring the predicted "bare" $R_{b\tau}^0(M_Z)$ values given in Table 3 into agreement with experiment

$$R_{b\tau}(M_Z) = R_{b\tau}^0(M_Z)[1 + \Delta] = R_{b\tau}^{\text{exp}}(M_Z) \quad (45)$$

(see $R_{b\tau}^{\text{exp}}(M_Z)$ above). Interestingly, as one can see from Table 3, the required values of Δ turn out to be in the range $\Delta = -0.3 \div -0.1$. The SUSY loop corrections in this range are readily obtained for most of SUSY parameter space [26] with the values of $\alpha_s(M_Z)$ and m_t in Table 3. So, presently, the bottom-tau unification prediction on its own does not seem to be a critical test of the $SU(7)$ model for either large or small $\tan\beta$.

Our results, obtained by numerical integration of all the RG equations listed above, are summarized in Figures (1, 2) and Tables (2, 3). The predicted $\alpha_s(M_Z)$ values are in a good agreement with the world average value (see Figure 1), in sharp contrast to the standard SUSY $SU(5)$ model predictions taken under the same conditions. Examples of string-scale unification are presented in Tables 2 and 3 for the small and large $\tan\beta$ cases, respectively. It is worth noting that the large $\tan\beta$ examples actually include "medium" values of $\tan\beta$ down to $\tan\beta \approx 20$, in contrast to the prototype $SU(5)$ model. Remarkably, string-scale unification appears to work both in

the case of a large bottom Yukawa coupling constant (the first line in Table 3), and in the case of top-bottom unification with a small common Yukawa coupling constant (the third line).

Thus we have seen how the missing VEV generating superpotential W_A (6) opens the way to a natural string-scale grand unification in supersymmetric $SU(7)$, prescribed at low energies by the gauge coupling values and the standard MSSM particle content plus one family of PG states (1). This is explicitly demonstrated in Figure 2.

At the same time, due to the reflection symmetry ($\Sigma \rightarrow -\Sigma$, $\Omega \rightarrow \Omega$) of the superpotential W_A , the Planck scale inherited smearing operators, which induce a Σ dependence into the kinetic terms of the SM gauge bosons, must have dimension 6 and higher,

$$\delta L = \frac{c}{M_P^2} Tr(GG\Sigma^2) + \dots \quad (46)$$

Here G is the gauge field-strength matrix of the $SU(7)$ model and c is some dimensionless constant of order 1. Thus, the influence of gravitational corrections on our gauge coupling predictions seems to be negligible, in contrast to the standard $SU(5)$ model predictions which can largely be smeared out by the dimension 5 operator $\frac{c'}{M_P} Tr(GG\Sigma)$ [9].

5 Conclusions

We have shown that a missing VEV vacuum configuration, which solves the doublet-triplet splitting problem and ensures the survival of the MSSM at low energies, only emerges in extended $SU(N)$ SUSY GUTs with $N \geq 7$. Furthermore, a realistic supersymmetric $SU(7)$ model was constructed which provides answers to many questions presently facing the prototype SUSY $SU(5)$ model: the doublet-triplet splitting problem, string scale unification, the hierarchy of baryon vs lepton number violation, quark and lepton (particularly neutrino) masses and mixings etc.

With the chosen assignment of matter and Higgs superfields in our $SU(7)$ model, the situation at low energies looks as if one had just the prototype $SU(5)$ as a starting symmetry, except that one family of PG states of type (1) appears when a missing VEV vacuum configuration develops in $SU(7)$. Apart from this exception, all other extra $SU(7)$ inherited states in matter

and Higgs multiplets acquire GUT scale masses during symmetry breaking, thus completely decoupling from low-energy physics. Another distinctive feature of our model concerns the relatively low mass scale of the adjoint moduli Σ_0 , Σ_3 and Σ_8 surviving after the $SU(7)$ breaks and, more importantly, their mass-splitting which inevitably appears in the missing VEV generating superpotential (6). The threshold corrections due to these states lead to a very different unification picture in $SU(7)$. String scale unification is obtained with a QCD coupling constant in agreement with the world average value, $\alpha_s(M_Z) = 0.119 \pm 0.002$, for both small and large $\tan \beta$.

There is no fundamental reason for exact R -parity (RP) symmetry in the framework of supersymmetric GUTs, where not only fermions but also their scalar superpartners are the natural carriers of lepton and baryon numbers. Accordingly, we constructed a general (RP -violating) superpotential where the effective lepton number violating couplings immediately evolve from the GUT scale. However the baryon number non-conserving ones are safely projected out by the missing VEV vacuum configuration which breaks the $SU(7)$ symmetry down to that of the MSSM. At the next stage when SUSY breaks, the radiative corrections shift the missing VEV components to some nonzero values of order M_{SUSY} , thereby inducing the ordinary Higgs doublet mass, on the one hand, and tiny baryon number violation, on the other. So, a missing VEV solution to the gauge hierarchy problem leads in fact to a similar hierarchy of baryon vs lepton number violation. Finally, an additional anomalous $U(1)_A$ symmetry introduced into the model purely as a missing VEV protecting symmetry was found to naturally act also as a family symmetry, which can lead to the observed pattern of quark and lepton masses and mixings.

To conclude the most crucial prediction of the presented $SU(7)$ model must surely be the very existence of the PG states and their superpartners of type (1). While at present not producing any unacceptable experimental predictions, they may include a few very long-lived states, depending on the details of their mixing pattern with the ordinary MSSM Higgs states. The colour-triplet states among them may gather together with ordinary quarks to give a new series of heavy hadrons (mesons and baryons), the lightest of which could be almost stable [27]. These PG states would clearly strongly influence particle phenomenology at the TeV scale.

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Figure captions

Figure 1: The $SU(7)$ predictions of $\alpha_s(M_Z)$ as a function of the grand unification scale M_U . Curves are shown for small $\tan\beta$ values (the dotted line) with top-Yukawa coupling $\alpha_t(M_U) = 0.1$ and the H -state mass threshold $M_* = 10^{16}$ GeV, and for large $\tan\beta$ values (the solid line) corresponding to $\alpha_t(M_U) = 0.02$, $\alpha_b(M_U) = 0.1$ and $M_* = 10^{16.8}$ GeV. The unification mass M_U varies from the MSSM unification point ($M_U^0 = 10^{16.3}$ GeV) to the string scale ($M_U = M_{str} = 10^{17.8}$ GeV for the Kac-Moody level $k = 2$), while the proton decay inducing colour-triplet mass is assumed to be at the unification scale in all cases. The all-shaded areas on the left are generally disallowed by the present bound [4] on nucleon stability for both cases, (small $\tan\beta$, dark) and (large $\tan\beta$, light) respectively.

Figure 2: The unification of the gauge coupling constants α_1 , α_2 and α_3 at the string scale is shown. The breaks on the curves correspond to the heavy

thresholds associated with the adjoint moduli Σ_3 and Σ_8 (on the left) and the H states (on the right).

Tables

Table 1: The $U(1)_A$ ($e^{iQ_A\theta}$) and Z_2 ($e^{in_Z\pi}$) charges, which allow just the couplings appearing in the total superpotential W_T (34).

\mathcal{F}	Ψ	$\bar{\Psi}$	ξ	$\bar{\zeta}$	H	\bar{H}	Σ	Ω	φ	$\bar{\varphi}$	η	$\bar{\eta}$	Φ	$\bar{\Phi}$	S
Q_A	0	0	-3	-1	-2	2	0	0	-2	3	2	1	-5	4	1
n_Z	1	0	0	0	0	1	1	0	0	1	0	1	0	0	0

Table 2: The 2-loop order string-scale unification for the small $\tan\beta$ case is presented. All masses are given in GeV. The top quark Yukawa coupling $\alpha_t(M_U)$ and H -state mass threshold $M_*(M_U)$ are taken as basic string scale input parameters ($M_U = M_{str} = \sqrt{8\pi\alpha_U} \cdot 5.27 \cdot 10^{17}$ for the Kac-Moody level $k = 2$), while low scale input parameters include $\alpha_{em}^{-1} = 127.9 \pm 0.1$, $\sin^2\theta_W = 0.2313 \pm 0.0002$ and the top quark pole mass $m_t = 175 \pm 6$. The values of $\alpha_s(M_Z)$, the $SU(7)$ unified coupling constant α_U , $\tan\beta$, the "bare" (uncorrected by gravitational contributions) bottom-tau mass ratio $R_{b\tau}^0(M_Z)$ and the adjoint moduli triplet Σ_3 mass $M_\Sigma(M_U)$ (the adjoint octet Σ_8 mass is $\frac{1}{2}M_\Sigma(M_U)$) are then predicted.

$\alpha_t(M_U)$	$M_*(M_U)$	$M_\Sigma(M_U)$	α_U	$\alpha_s(M_Z)$	$\tan\beta$	$R_{b\tau}^0(M_Z)$
0.1	10^{16}	$10^{13.3}$	0.080	0.120	1.6	2.2
0.08	$10^{15.8}$	$10^{13.1}$	0.082	0.119	1.6	2.3
0.04	$10^{15.4}$	$10^{12.9}$	0.087	0.119	1.8	2.4
0.02	$10^{15.2}$	$10^{12.8}$	0.090	0.119	2.2	2.6
0.01	10^{15}	$10^{12.8}$	0.092	0.119	4.8	2.7

Table 3: The 2-loop order string-scale unification for the large $\tan\beta$ case is presented. All masses are given in GeV. The Yukawa couplings $\alpha_{t,b,\tau}(M_U)$

and H -state mass threshold $M_*(M_U)$ are taken as basic string scale input parameters ($M_U = M_{str} = \sqrt{8\pi\alpha_U} \cdot 5.27 \cdot 10^{17}$ for the Kac-Moody level $k = 2$), while low scale input parameters include $\alpha_{em}^{-1} = 127.9 \pm 0.1$ and $\sin^2 \theta_W = 0.2313 \pm 0.0002$. The values of $\alpha_s(M_Z)$, the $SU(7)$ unified coupling constant α_U , $\tan \beta$, the top quark pole mass m_t , the "bare" (uncorrected by SUSY loop contributions) bottom-tau mass ratio $R_{b\tau}^0(M_Z)$ and the adjoint moduli triplet Σ_3 mass $M_\Sigma(M_U)$ (the adjoint octet Σ_8 mass is $\frac{1}{2}M_\Sigma(M_U)$) are then predicted.

$\alpha_t(M_U)$	$\alpha_{b,\tau}(M_U)$	$M_*(M_U)$	$M_\Sigma(M_U)$	α_U	$\alpha_s(M_Z)$	m_t	$\tan \beta$	$R_{b\tau}^0(M_Z)$
0.025	0.3	$10^{17.8}$	10^{14}	0.066	0.120	176	55.1	1.9
0.02	0.1	$10^{16.8}$	$10^{13.5}$	0.073	0.119	175	52.8	1.9
0.01	0.01	$10^{15.2}$	$10^{12.7}$	0.089	0.118	172	38.9	2.3
0.01	0.004	$10^{15.2}$	$10^{12.8}$	0.089	0.119	175	30.3	2.4
0.01	0.001	$10^{15.2}$	$10^{12.9}$	0.090	0.120	179	17.9	2.6

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